FULLY WORKED SOLUTIONS

Context 3: The physics of sport

Chapter 8: The drop zone

Chapter questions

1. $F_{drag} = ½C_D\rho v^2A = ½ \times 1.6 \times 0.9 \times (6)^2 \times 4 = 103.7$ N

2. $F_{net} = w – F_{drag} = (2 \times 9.8) – 3.7 = 15.9$ N

   $a = F_{net}/m = 15.9/2 = 7.95$ m s^{-2}

3. $u = 0$, $a = –9.8$ m s^{-2} , $s = –5$ m

   $v^2 = u^2 + 2as = 0 + (2 \times – 9.8 \times – 5) = 98$

   $v = –9.9$ m s^{-1}

4. $u = 0$, $v = –6.2$ m s^{-1}, $a = –9.8$ m s^{-2}

   $(–6.2)^2 = u^2 + 2as = 0 + (2 \times – 9.8)s$

   $38.4 = –19.6s$

   $s = –2$ m

5. $u = –2$ m s^{-1}, $s = –5$ m, $a = –9.8$ m s^{-2}

   $v^2 = u^2 + 2as = (–2)^2 + (2 \times – 9.8 \times – 5)$

   $v^2 = 102$

   $v = –10$ m s^{-1}

6. $k = mg/\Delta x = (70 \times 9.8)/10 = 69$ N m^{-1}

7. $F = kx = 20 \times 120 = 2\,400$ N

8. $k = mg/\Delta x = (75 \times 9.8)/10 = 73.5$ N m^{-1}

   With 10% error, $73.5 + 0.1 \times 73.5 = 80.8$ N m^{-1}

9. $v_x = u_x = 900$ m s^{-1}

   $v_y = u_y + at = 0 + (–9.8 \times 0.8) = –7.84$ m s^{-1}

10. $v_x = u_x = 5$ m s^{-1}

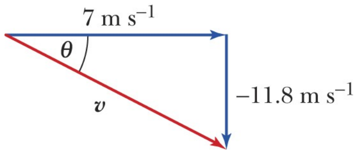
    $v_y = u_y + at = 0 + (–9.8 \times 0.5) = –4.9$ m s^{-1}

11. $v_x = u_x = 7$ m s^{-1}

    $v_y = u_y +at = 0 + (–9.8 \times 1.2) = –11.8$ m s^{-1}

    $v^2 = v_x^2 + v_y^2 = 7^2 + (–11.8)^2$

    $v = 13.7$ m s^{-1}



$\theta = \tan^{-1}(11.8/7) = 59.3°$

Therefore, the velocity is 13.7 m s^{-1} at 59.3° to the horizontal.

12. $x = u_x t$

$t = x/u_x = 300/800 = 0.4$ s

13. $y = u_y t + ½at^2$

$–0.3 = 0 + ½x – 9.8 \times t^2$

$–0.3 = –4.9t^2$

$t = 0.247$ s

$x = u_x t = 50 \times 0.247 = 12.4$ m

14. $y = u_y t + ½at^2$

$–1.5 = 0 + ½ \times –9.8 \times t^2$

$–1.5 = –4.9t^2$

$t = 0.55$ s

$x = u_x t = 7 \times 0.55 = 3.9$ m

15. $u_x = 22 \cos 30° = 19$ m s^{-1}

$u_y = 22 \sin 30° = 11$ m s^{-1}

(a) $y = 0$

$y = u_y t + ½at^2$

$0 = 11\text{ t} + ½ \times – 9.8 \times t^2$

$0 = 11\text{ t} – 4.9t^2$

$0 = 11 – 4.9t$

$4.9t = 11$

$t = 2.24$ s

(b) $x = u_x t = 19 \times 2.24 = 42.6$ m

(c) $y = u_y t + ½ at^2$

$y = (11 \times 0.6) + ½ \times – 9.8 \times (0.6)^2$

$y = 6.6 – 1.8 = 4.8$ m

16. $u = 50$ km $h^{-1} = 13.9$ m s^{-1}

$u_x = 13.9 \cos 68° = 5.2$ m s^{-1}

$u_y = 13.9 \sin 68° = 12.9$ m s^{-1}

When caught, $y = 0$

$y = u_y t + ½at^2$

$0 = 12.9\ \mathrm{t} - 4.9t^2$

$0 = 12.9 - 4.9t$

$4.9t = 12.9$

$t = 2.63$ s

$x = u_x t = 5.2 \times 2.63 = 13.7$ m

17. $u_x = u \cos 40° = 0.77u$

$u_y = u \sin 40° = 0.64u$

$x = u_x t$

$350 = 0.77ut$

$ut = 454.5$ (1)

When it reaches the ground, $y = 0$.

$y = u_y t + ½at^2$

$0 = 0.64ut - 4.9t^2$ (2)

Substituting (1) into (2) gives:

$0 = 0.64 \times (454.5) - 4.9t^2$

$0 = 291 - 4.9t^2$

$4.9t^2 = 291$

$t = 7.7$ s

As $ut = 454.5$,

$u = 454.5/t$

$u = 454.5/7.7 = 59\ \mathrm{m\ s^{-1}}$

Review questions

6. $r = 0.1$ m, $C_D = 1.7$, $\rho = 1.2\ \mathrm{kg\ m^{-3}}$, $v = 4\ \mathrm{m\ s^{-1}}$

$A = \pi r^2 = \pi(0.1)^2 = 0.031\ \mathrm{m^2}$

$$\boldsymbol{F}_{\mathrm{drag}} = \frac{1}{2} C_D \rho v^2 A = \frac{1}{2} \times 1.7 \times 1.2 \times 4^2 \times 0.031 = 0.51\ \mathrm{N}$$

7. When terminal velocity is reached, drag will be equal to weight. Therefore:

$\boldsymbol{F}_{\mathrm{drag}} = \boldsymbol{w} = m\boldsymbol{g} = 80 \times 9.8 = 784$ N

8. (a) $u_y = 0$, $y = -6$ m, $a = -9.8\ \mathrm{m\ s^{-2}}$

$$y = \mathrm{u}_y t + \frac{1}{2} at^2$$

$-6 = 0 + \frac{1}{2}(-9.8)t^2$

$-6 = -4.9t^2$

$t^2 = 1.22$

$t = 1.1$ s

(b) $u_y = 0$, a = –9.8 m s^{-2}, $t = 1.1$ s

$v_y = u_y + at$

$= 0 + (-9.8) \times 1.1$

$= -10.8$ m s^{-1}

(c) $u_y = 0$, $t = 0.1$ s, $a = -9.8$ m s^{-2}

$y = u_y t + \frac{1}{2}at^2$

$= 0 + \frac{1}{2}(-9.8)(0.1)^2$

$= 0.049$ m

Distance above the water = 6 m – 0.049 m = 5.95 m

9. $m = 180$ kg, $\boldsymbol{g} = -9.8$ m s^{-2}, $y = -46$ m, k = 120 N m^{-1}

If $\boldsymbol{x}$ is the original length of the bungee and $\Delta\boldsymbol{x}$ is the change in length of the bungee, then $\boldsymbol{x} + \Delta\boldsymbol{x} \leq 46$ m for safety.

At the bottom of the bungee jump, weight will be the force extending the bungee, so:

$\boldsymbol{w} = \boldsymbol{F}_{\text{elastic}}$

$m\boldsymbol{g} = k\Delta\boldsymbol{x}$

$\Delta x = \frac{mg}{k}$

$x + \frac{mg}{k} \leq 46$

$x + \frac{180 \times 9.8}{120} \leq 46$

$x + 14.7 \leq 46$

$x \leq 46 - 14.7$

$x \leq 31.3\ m$

10. $x = 200$ m, $y = -1$ m, $u_y = 0$, $a = -9.8$ m s^{-2}

$y = u_y t + \frac{1}{2}a t^2$

$-1 = \frac{1}{2}(-9.8)t^2$

$t^2 = 0.20$

$t = 0.45$ s

$u_x = \frac{x}{t} = \frac{200}{0.45} = 444\ \text{m s}^{-1}$

11. (a) $y = 10$ m, $a = -9.8$ m s^{-2}, $v_y = 0$

$$v_y^2 = u_y^2 + 2ay$$
$$0 = u_y^2 + 2(-9.8)(10)$$
$$0 = u_y^2 - 196$$
$$196 = u_y^2$$
$$u_y = 14\ \text{m s}^{-1}$$

(b) $y = -10$ m, $a = -9.8$ m s^{-2}, $u_y = -3$ m s^{-1}

$$v_y^2 = u_y^2 + 2ay$$
$$= (-3)^2 + 2(-9.8)(-10)$$
$$= 9 + 196$$
$$= 205$$
$$v_y = \sqrt{205} = -14.3\ \text{m s}^{-1}$$

12. $u_x = 15$ m s^{-1}, $x = 6$ m, $a = -9.8$ m s^{-2}, $u_y = 0$

$t = \frac{x}{u_x} = \frac{6}{15} = 0.4$ s

$y = u_y t + \frac{1}{2}at^2$

$= 0 + \frac{1}{2}(-9.8)(0.4)^2$

$= -0.78$ m

13. $x = 4.3$ m, $\theta = 12°$, $y = 0$

$x = u_x t$

$4.3 = (u \cos 12°)t$

$4.3 = 0.98ut$

$ut = 4.4$ (1)

$y = u_y t + \frac{1}{2}at^2$

$0 = (u \sin 12°)t + \frac{1}{2}(-9.8)t^2$

$0 = 0.21ut - 4.9t^2$

Substituting equation (1) into this equation gives:

$0 = 0.21 \times 4.4 - 4.9t^2$

$4.9t^2 = 0.924$

$t^2 = 0.19$

$t = 0.44$ s

As $4.4 = ut$ from (1), then

$u = \frac{4.4}{t} = \frac{4.4}{0.44} = 10 \text{ m s}^{-1}$

14. $x = 2.5$ m, $t = 4$ s, $u = 5$ m s^{-1}

(a) $x = u_x t$

$x = (u\cos\theta)t$

$2.5 = 5 \cos\theta \times 4$

$2.5 = 20 \cos\theta$

$\cos\theta = \frac{2.5}{20} = 0.125$

$\theta = 82.8°$

(b) At maximum height, $v_y = 0$

$u_y = u \sin\theta = 5 \sin 82.8° = 4.96$ m s^{-1}

$v_y^2 = u_y^2 + 2ay$

$0 = (4.96)^2 + 2(-9.8)y$

$0 = 24.6 - 19.6y$

$19.6y = 24.6$

$y = 1.3$ m

15. $x = 300$ m, $u = 600$ m s^{-1}, $y = 0$

$u_x = 600 \cos\theta$

$u_y = 600 \sin\theta$

$x = u_x t$

$300 = 600t \cos\theta$

$t = \frac{300}{600 \cos\theta}$

$t = \frac{1}{2 \cos\theta}$ (1)

$y = u_y t + \frac{1}{2} a t^2$

$0 = 600 \sin\theta t + \frac{1}{2}(-9.8)t^2$

$0 = 600 \sin\theta - 4.9t$

$4.9t = 600 \sin\theta$

Substitute equation (1) for t to get:

$$4.9\left(\frac{1}{2\cos\theta}\right) = 600\sin\theta$$

$$4.9 = 1200\cos\theta\sin\theta$$

$$4.08 \times 10^{-3} = \frac{1}{2}\sin 2\theta$$

$$\sin 2\theta = 8.2 \times 10^{-3}$$

$$\theta = 0.23°$$

To find the height h that she was aiming above the target,

$$\tan(0.23°) = \frac{h}{300}$$

$$h = 1.22 \text{ m}$$

Therefore, the shooter aims at a point 1.22 m above the target.

16. $y = 0$, $t = 2.1$ s, $a = -9.8$ m s^{-2}

$$y = u_y t + \frac{1}{2}at^2$$

$$0 = u_y(2.1) + \frac{1}{2}(-9.8)(2.1)^2$$

$$0 = 2.1u_y - 21.6$$

$$21.6 = 2.1u_y$$

$$u_y = 10.3 \text{ m s}^{-1}$$

17. $u = 80$ km h^{-1} = 22.2 m s^{-1}, $\theta = 20°$, $y = -1$ m

$$u_x = 22.2\cos 20° = 20.9 \text{ m s}^{-1}$$

$$u_y = 22.2\sin 20° = 7.6 \text{ m s}^{-1}$$

$$y = u_y t + \frac{1}{2}at^2$$

$$-1 = 7.6t + \frac{1}{2}(-9.8)t^2$$

$$-1 = 7.6t - 4.9t^2$$

$$4.9t^2 - 7.6t - 1 = 0$$

Solve for t using the quadratic solution:

$$t = \frac{+7.6 \pm\sqrt{(-7.6)^2 - 4(4.9)(-1)}}{2 \times 4.9}$$

$$t = -0.12 \text{ s or } 1.86 \text{ s}$$

Obviously, the bike will land on the ramp when $t = 1.86$ s.

Let x represent the distance of the landing ramp from the last bus.

$x = 2 + 24 + x = 26 + x$

$26 + x = u_x t$

$26 + x = 20.9 \times 1.86 = 38.9$ m

$x = 38.9 - 26 = 12.9$ m

18. $u = 20$ m s^{-1}, $\theta = 30°$, $y = 1.2$ m

$u_x = 20 \cos 30° = 17.3$ m s^{-1}

$u_y = 20 \sin 30° = 10$ m s^{-1}

$y = u_y t + \frac{1}{2} a t^2$

$1.2 = 10t - 4.9t^2$

$4.9t^2 - 10t + 1.2 = 0$

Solve for t using the quadratic solution:

$$t = \frac{+10 \pm \sqrt{(-10)^2 - 4(4.9)(1.2)}}{2 \times 4.9}$$

$t = 0.13$ s or 1.9s

The first of these values represents when the ball was on its way up while the other is when the ball is coming down again. This is the value that we need to use.

$x_{\text{ball}} = u_x t = 17.3 \times 1.9 = 32.9$ m

$d_{\text{fielder}} = v_{\text{av}} t = 6 \times 1.9 = 11.4$ m

Separation between batter and fielder = 32.9 + 11.4 = 44.3 m